Thermal Noise and Design of the GQuEST Interferometer

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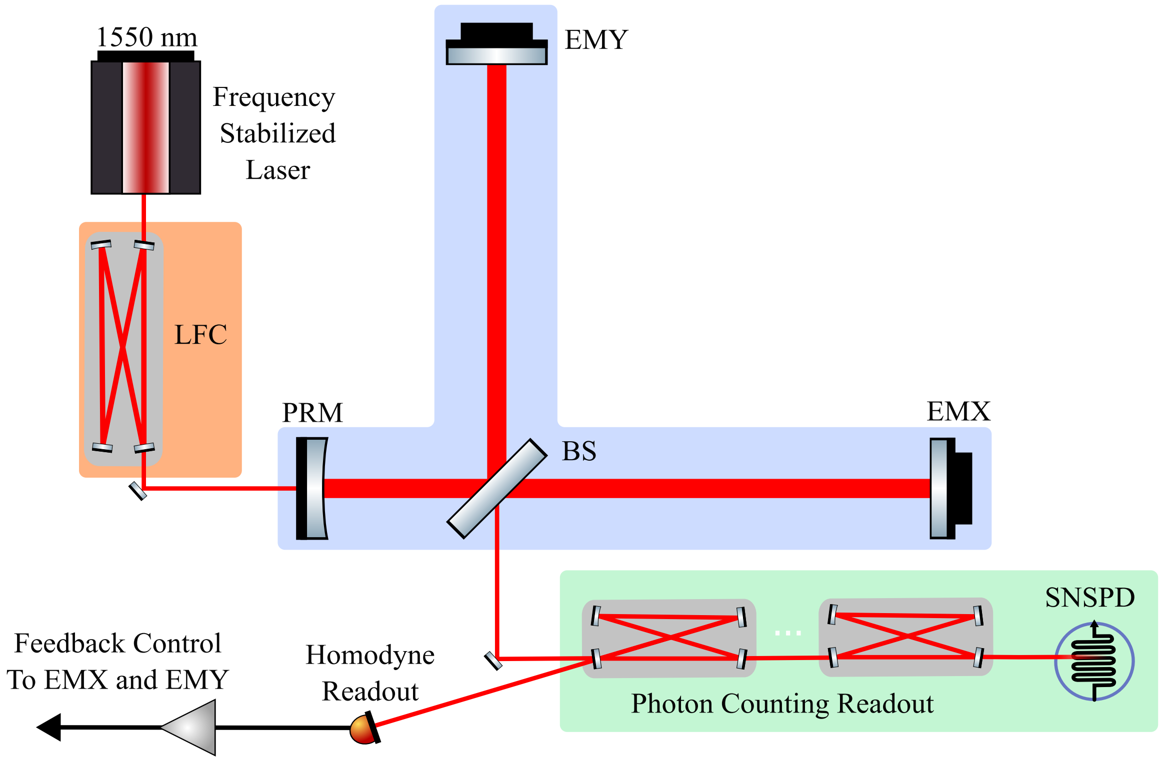
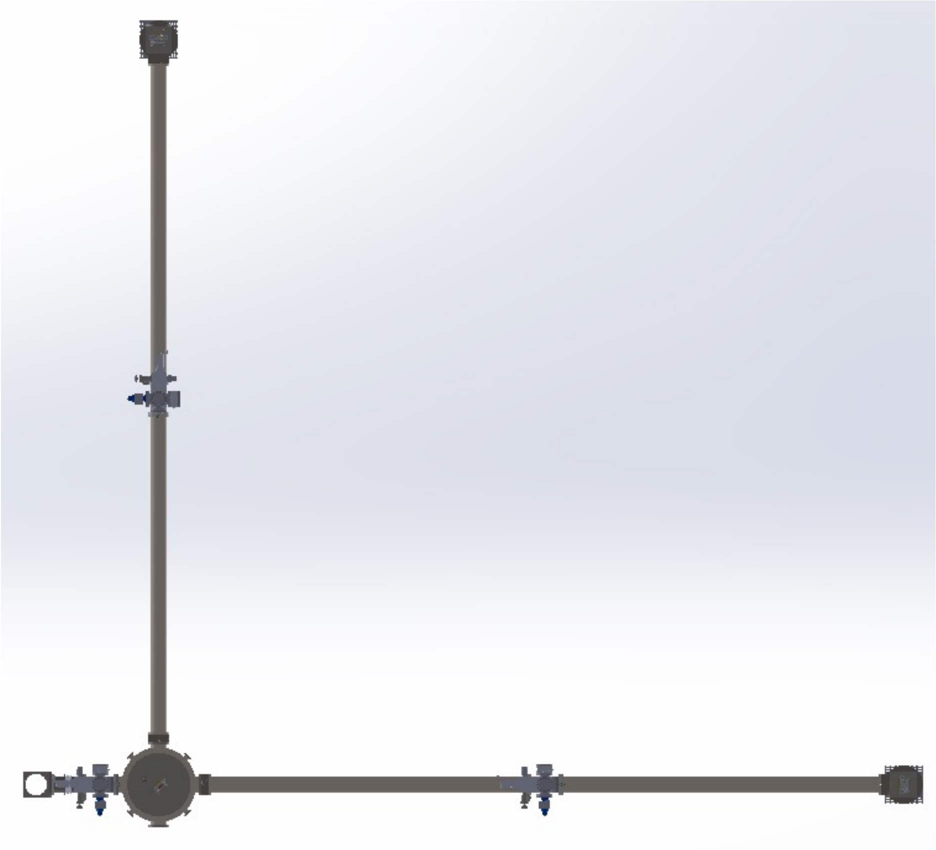
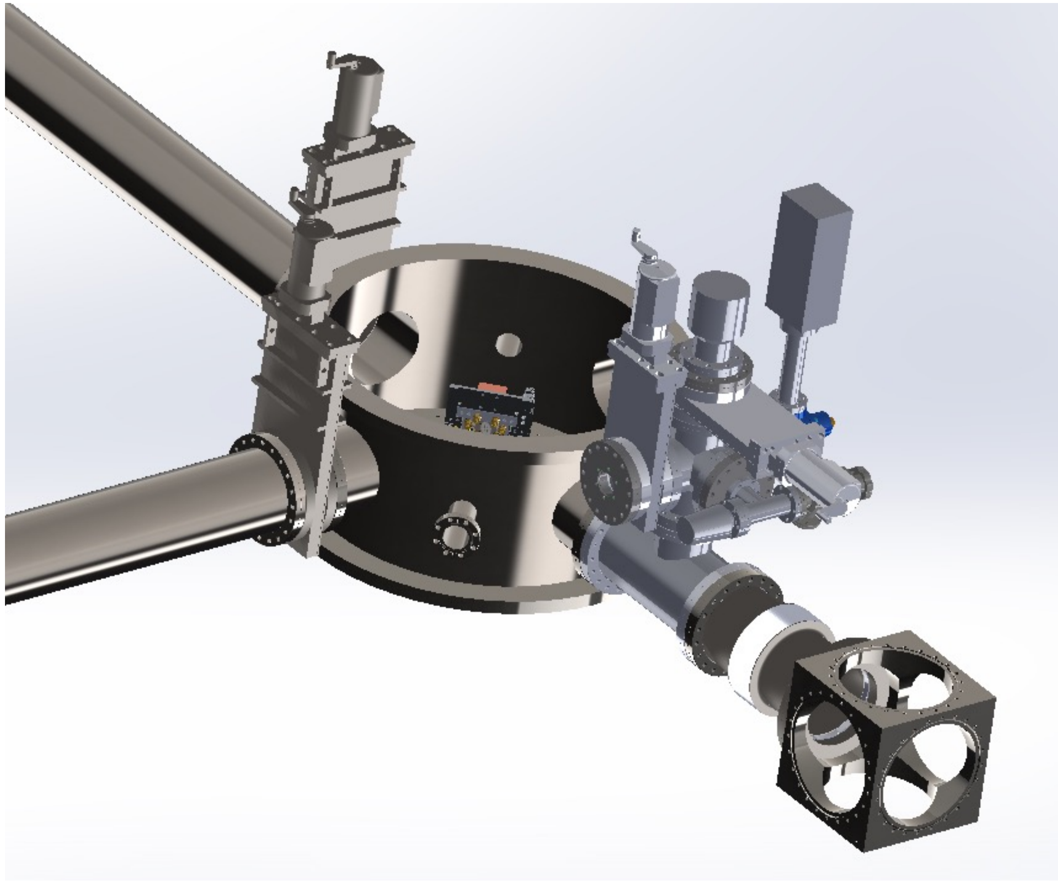
GQuEST Setup

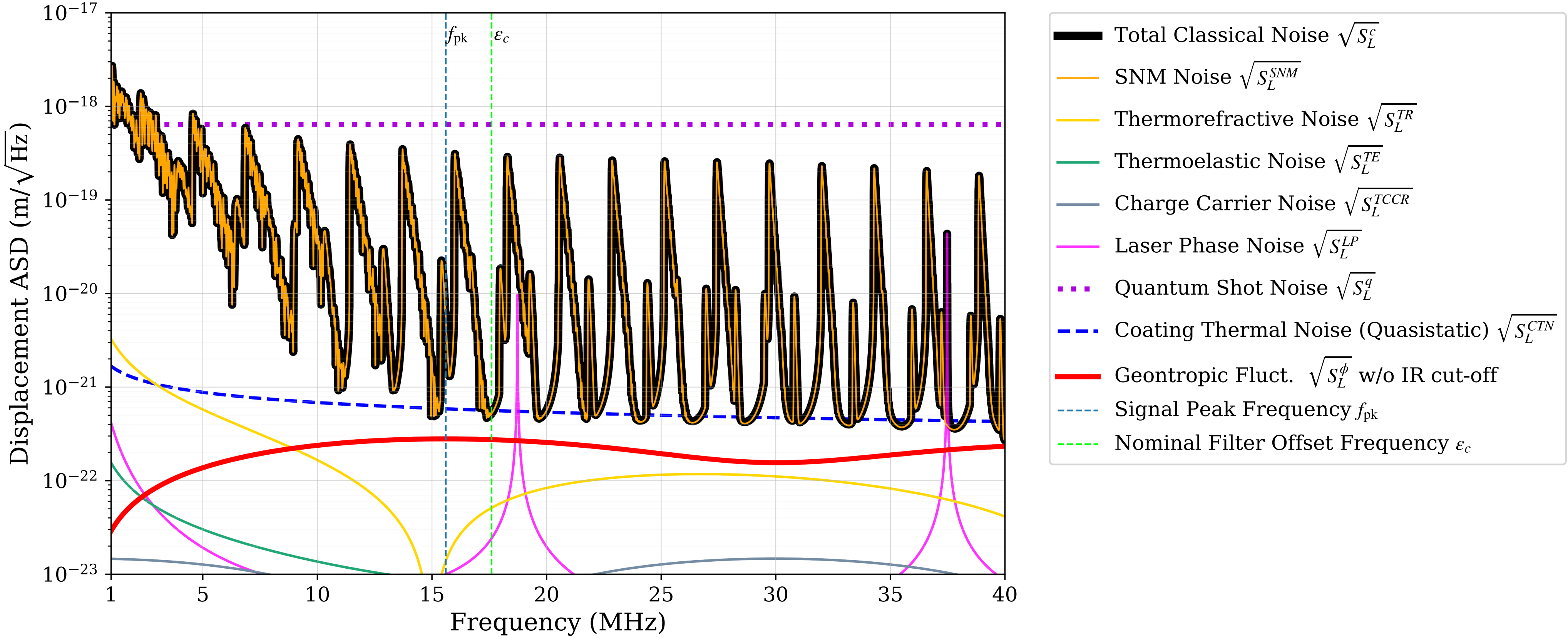


Actual Design



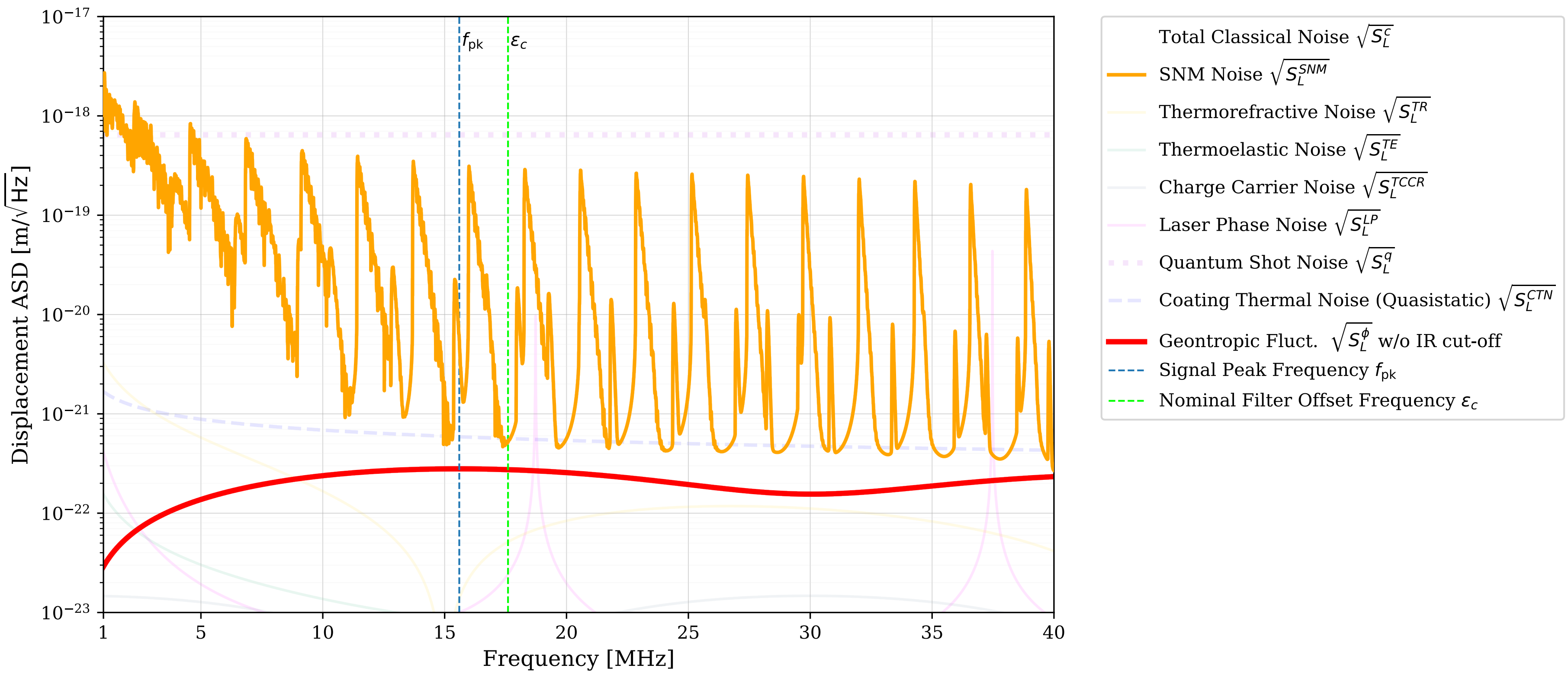


Noise Budget



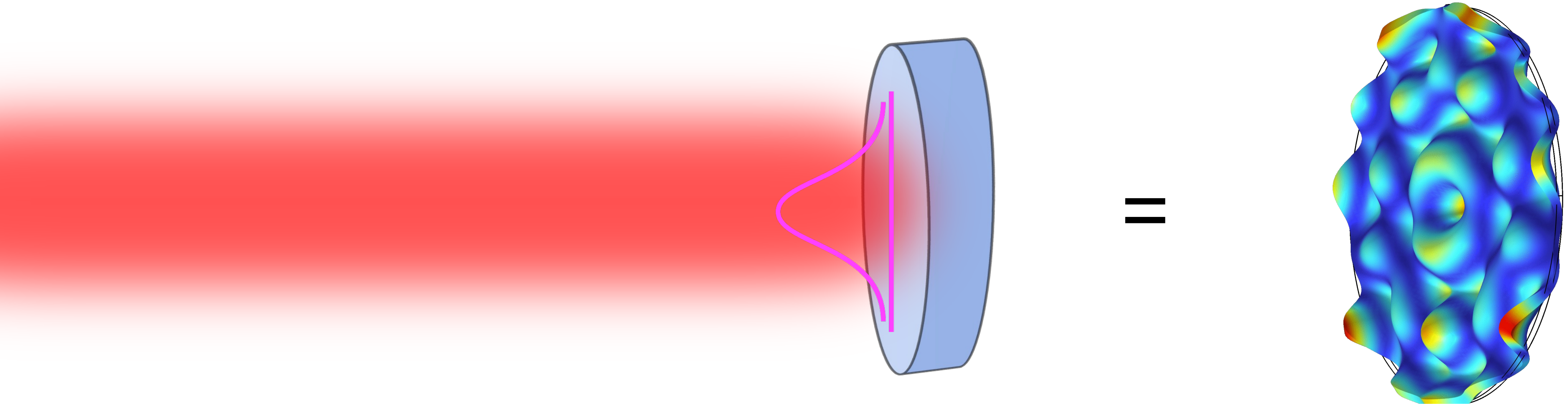
- This “noise budget” contains the dominant noise sources in the GQuEST interferometer
- We are dominated by bulk thermal noise from the optics

Solid Normal Mode Noise



- Each mirror has tiny vibrations, which change the measured length of the arm
- 2 ways of calculating the noise: susceptibility method and eigenmode decomposition

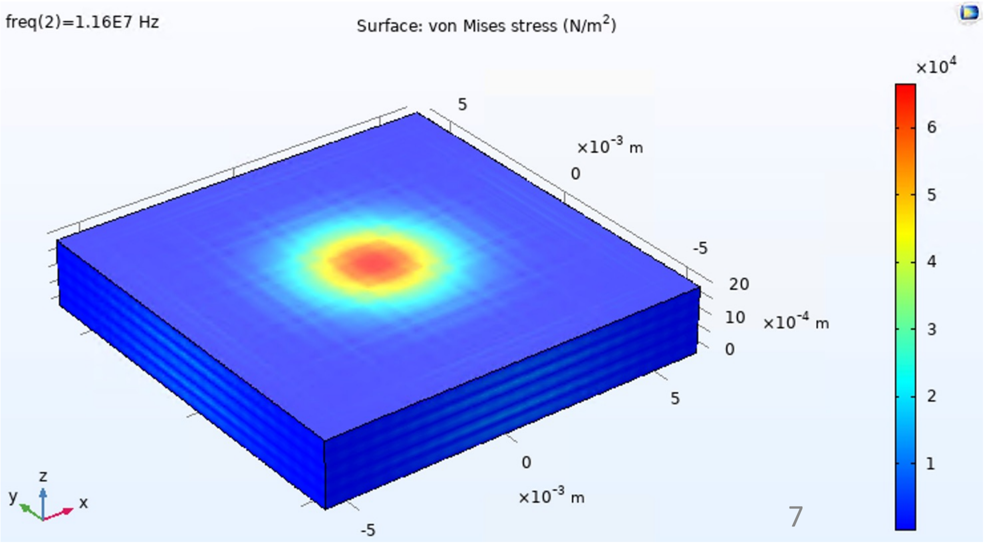
Solid Normal Modes



Susceptibility Method

- Solid Normal Noise can be modeled by treating the laser as a force and considering the power dissipated, “susceptibility method”
- This relies on the fluctuation-dissipation and equipartition theorems
- Uses the “admittance” from the laser beam modeled as a force
- Easier computationally, but less direct

$$S_L^{\mathrm{FDT}}(\Omega) = \frac{4k_BT}{\Omega}\frac{U_{\max}}{QF_{\mathrm{pk}}^2} = \frac{4k_BT}{\Omega^2}|\Re[Y(\Omega)]|$$

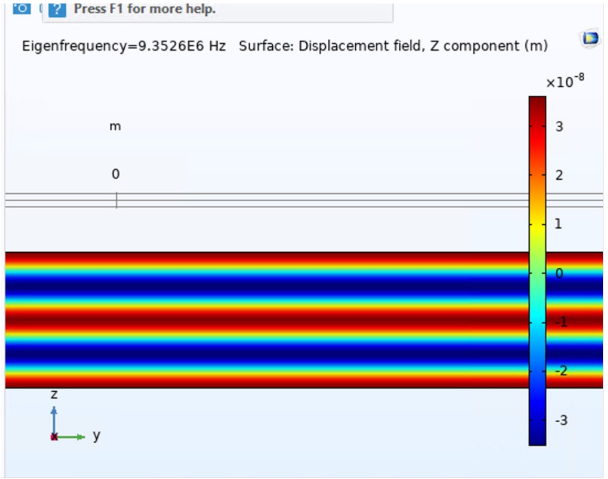


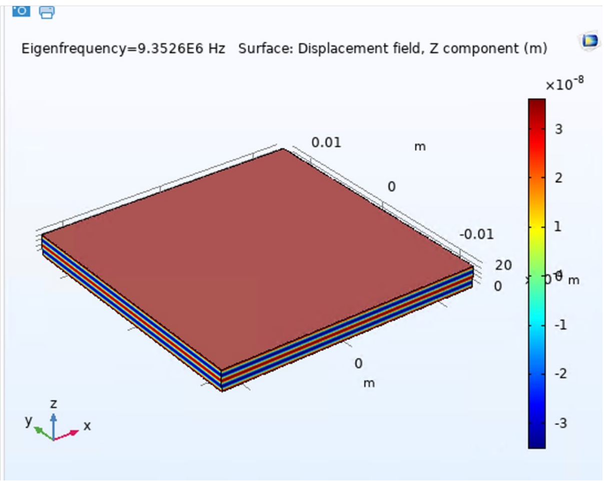
More on the Susceptibility Method

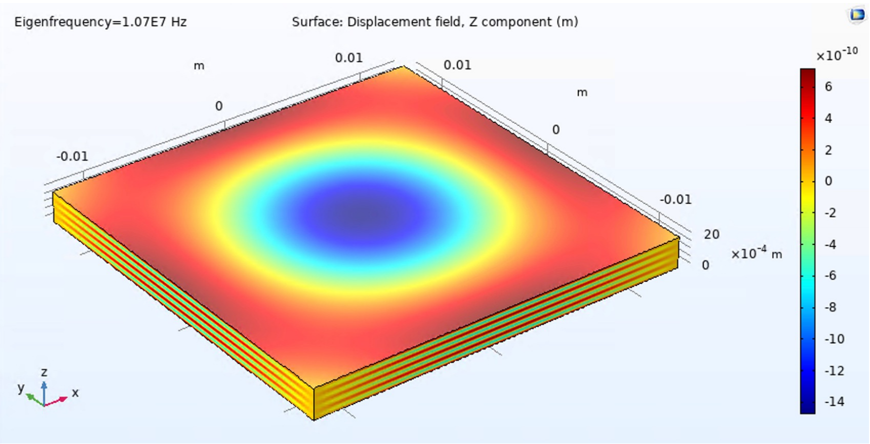
- Admittance = 1/impedance
- The real part is the dissipative term, like a resistance
- This is similar to Johnson-Nyquist noise

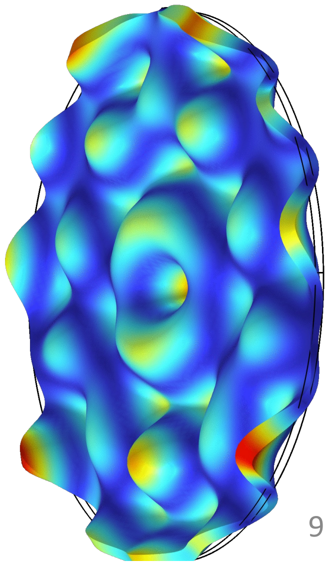
Eigenmode Decomposition Theory

- The mirror has (many) eigenmodes
- Each eigenmode displaces the mirror surface, which affects the phase of the light and looks like signal
- The strength of the noise from each eigenmodes is proportional to the overlap integral of the beam and the eigenmode
- Equipartition Theorem used here as well

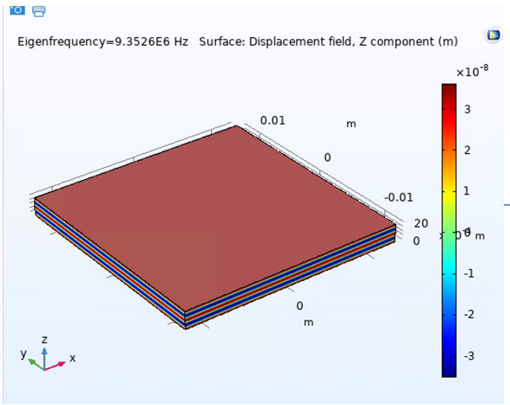




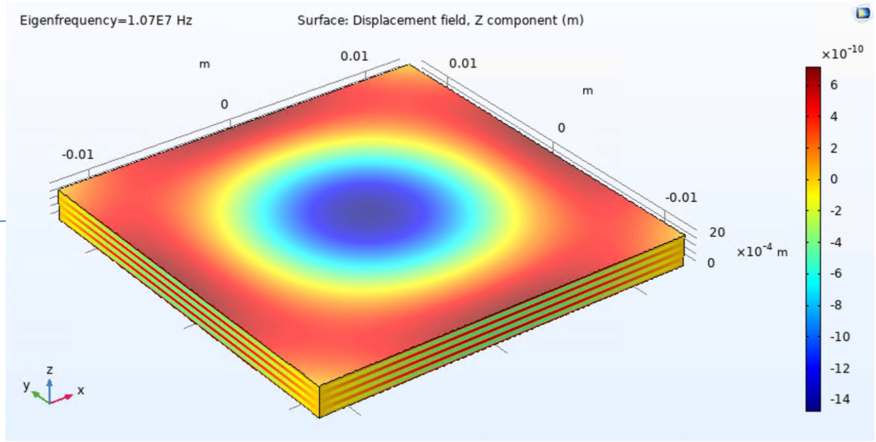




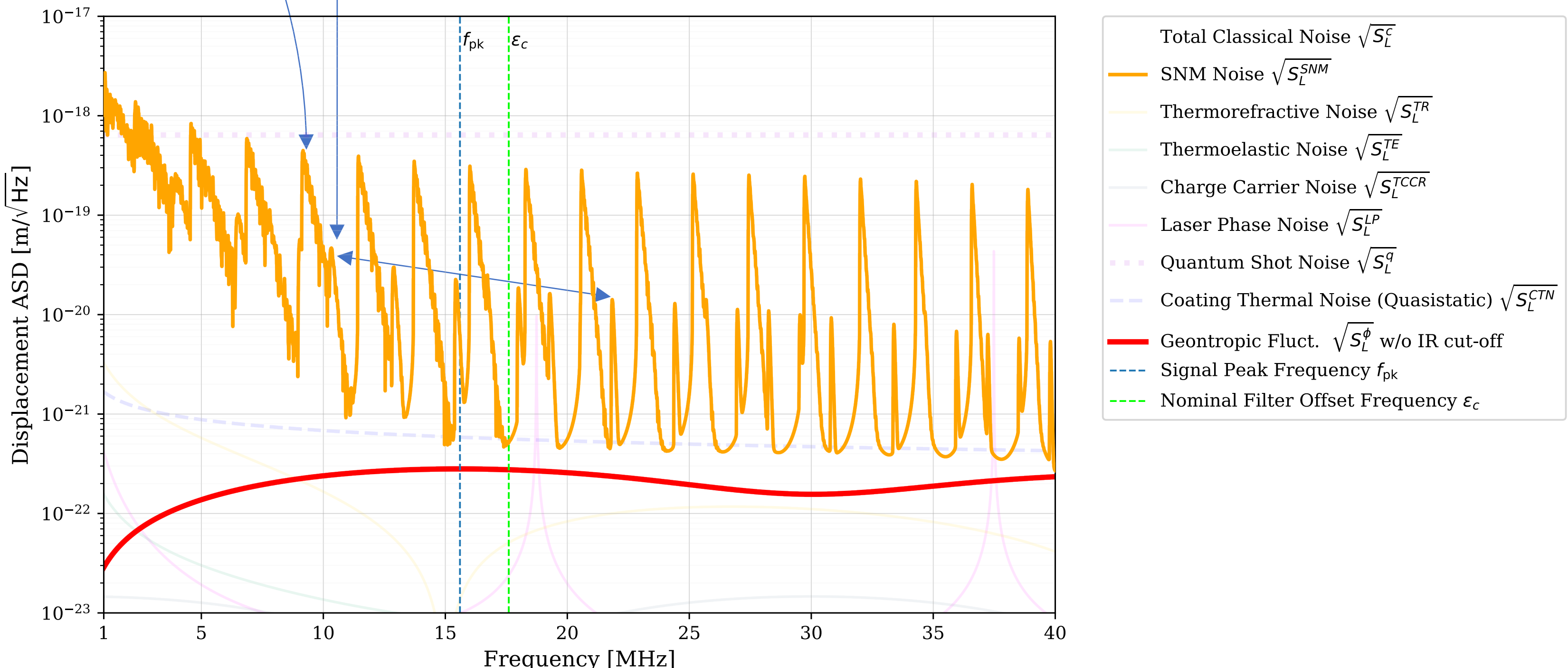
What are these modes?



Longitudinal (scalar potential)



Transverse (vector potential)



Eigenmode Decomposition Math

κ indexes over both mode types and all 3 axes

$$S_L^{\mathrm{SNM}}(\Omega,\kappa) = C_\kappa \frac{4k_{\mathrm{B}}T}{m\Omega\left(\omega_\kappa\right)^2}\left(\frac{Q_{\mathrm{B}}}{1+Q_{\mathrm{B}}^2\left(\left(\Omega/\omega_\kappa\right)^2-1\right)^2}\right)$$

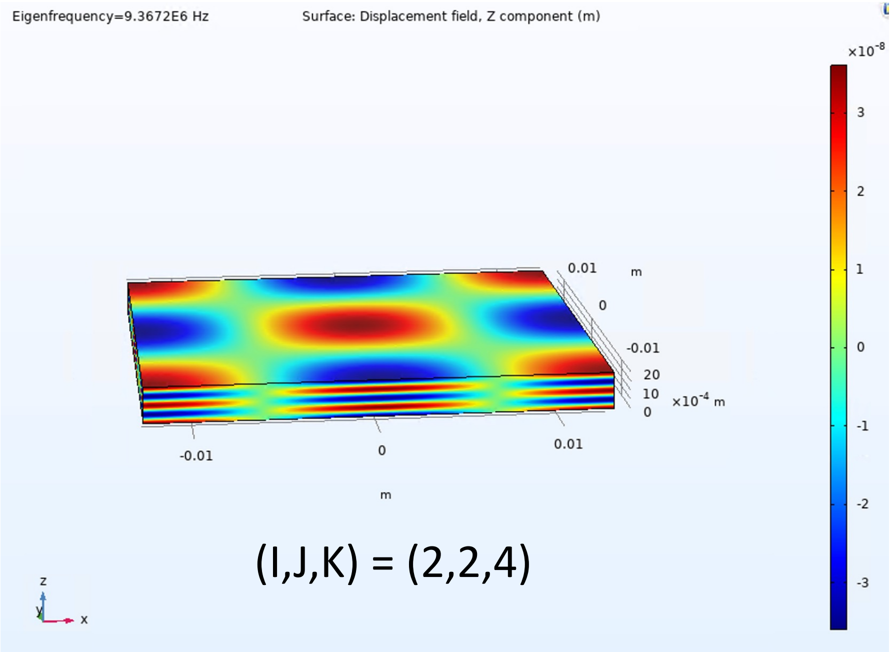
$$S_L^{\mathrm{SNM}}(\Omega) = \sum_\kappa S_L^{\mathrm{SNM}}(\Omega,\kappa)$$

$$S_L^{\mathrm{SNM}}(\Omega) = \sum_\kappa C_\kappa \frac{4k_{\mathrm{B}}T}{m\Omega\left(\omega_\kappa\right)^2}\left(\frac{Q_{\mathrm{B}}}{1+Q_{\mathrm{B}}^2\left(\left(\Omega/\omega_\kappa\right)^2-1\right)^2}\right)$$

What is the coupling for longitudinal modes?

$$C_\kappa = \frac{m\omega_\kappa^2 \Delta l_\kappa^2}{2V}$$

$$\Delta l_\kappa = A_L(z \cdot \boldsymbol{n}_L) \iint_S \cos\left(\frac{\pi I x}{s}\right) \cos\left(\frac{\pi J y}{s}\right) \frac{2}{\pi w^2} e^{-2\frac{(x-s/2)^2+(y-s/2)^2}{w^2}} dxdy$$



(I,J,K) = (2,2,4)

$$k_x = \pi I/s,\ k_y = \pi J/s,\ k_z = \pi K/h$$

$$C_\kappa = \frac{2K^2}{(I^2+J^2)(h^2/s^2)+K^2} e^{-\pi^2 w^2 (I^2+J^2)/(4s^2)}$$

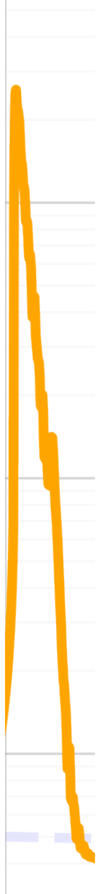
Analytic Estimation for constant K

$$S_L^{\mathrm{SNM}}(\Omega) \approx \sum_\kappa C_\kappa \frac{4k_{\mathrm{B}}T}{m\Omega\,(\omega_0)^2}\left(\frac{Q_{\mathrm{B}}}{1+Q_{\mathrm{B}}^2\left((\Omega/\omega_0)^2-1\right)^2}\right)$$

$$S_L^{\mathrm{SNM}}(\Omega) \approx \frac{4k_{\mathrm{B}}T}{m\Omega\,(\omega_0)^2}\left(\frac{Q_{\mathrm{B}}}{1+Q_{\mathrm{B}}^2\left((\Omega/\omega_0)^2-1\right)^2}\right)\sum_\kappa C_\kappa$$

$$\sum_{I,J} \frac{2K^2}{(I^2+J^2)(h^2/s^2)+K^2} e^{-\pi^2 w^2 (I^2+J^2)/(4s^2)}$$

$$\approx \iint \frac{2K^2}{(I^2+J^2)(h^2/s^2)+K^2} e^{-\pi^2 w^2 (I^2+J^2)/(4s^2)} dI\,dJ$$



Final Analytic Estimation

- Consider noise half-way between resonances, which is where we will look for the signal
- Evaluating the integral from the last slide and doing some algebra yields the following

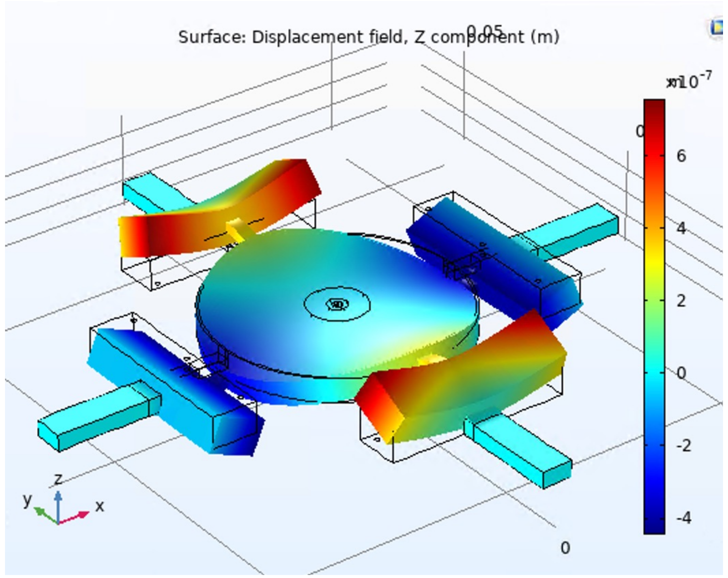
$$S_{L,min}^{BAW}(\Omega) = \frac{16 k_{\mathrm{B}} T h}{\pi^3 Q_B Y_s w^2 \Omega}$$

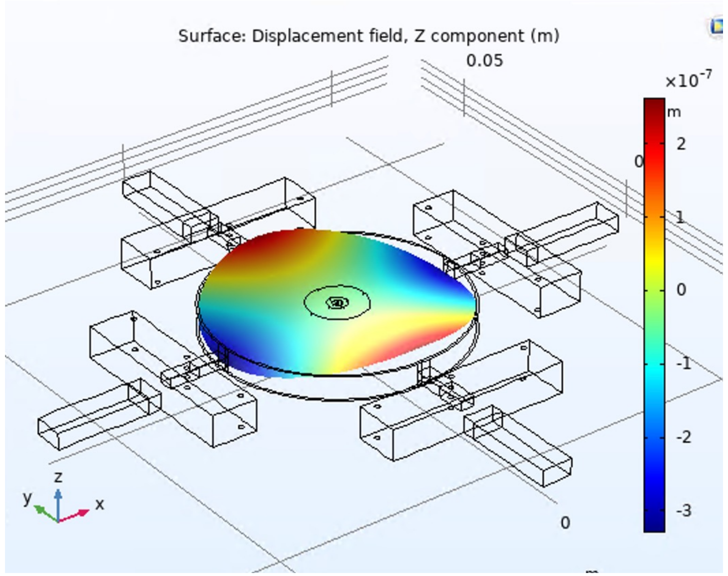
Limit 1 on SNM minimization

$$S_{L,min}^{BAW}(\Omega) = \frac{16 k_{\mathrm{B}} T h}{\pi^3 Q_B Y_s w^2 \Omega}$$

- Optic cannot be too thin, or coating stress becomes too problematic

$$r_{\mathrm{curv}} \approx \frac{1}{6}\left(\frac{Y_s h^2}{\sigma h_c (1 - v_s)}\right), \quad D \approx 12\left(\frac{\sigma h_c (1 - v_s)}{Y_s h^2}\right)$$

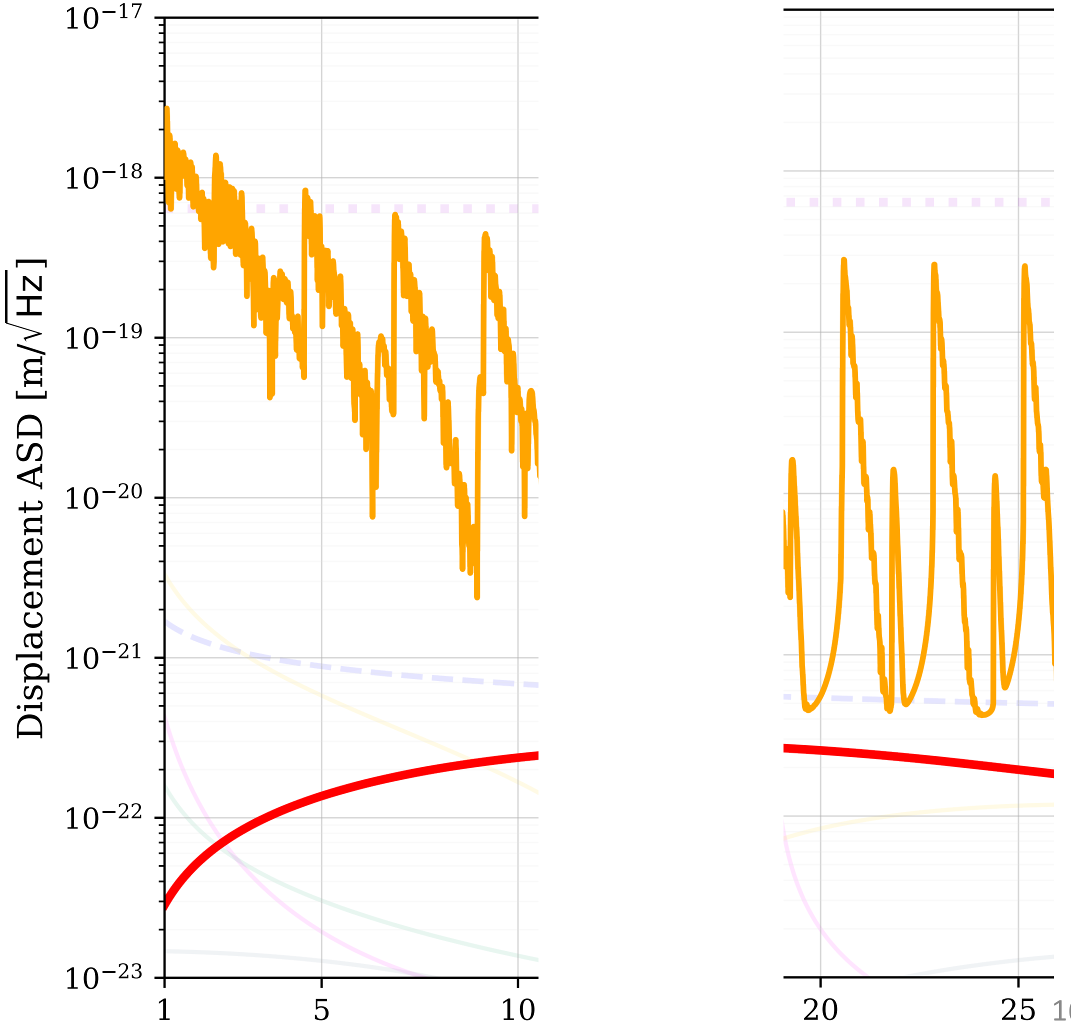




Limit 2 on SNM minimization

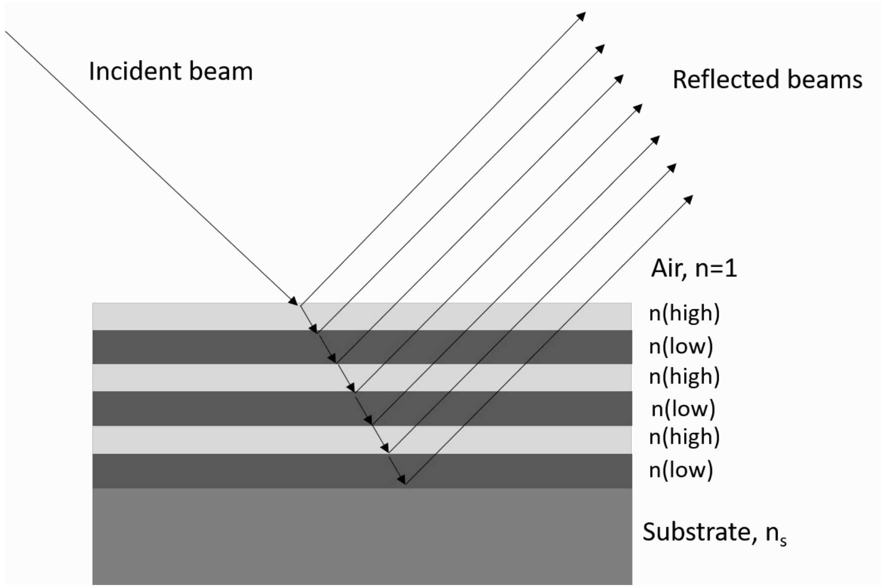
$$S^{BAW}_{L,min}(\Omega) = \frac{16 k_{\mathrm{B}} T h}{\pi^3 Q_B Y_s w^2 \Omega}$$

- Optic cannot be too stiff, or modes with transverse components couple in at higher frequencies



Coating Thermal Noise

- Need a coating on an optic to make it reflective
- Often very thin and made up of alternating layers, a “Bragg coating”
- Brownian noise is due to mechanical fluctuations, just like the bulk thermal noise, but the coating layer is much thinner so this noise presents differently



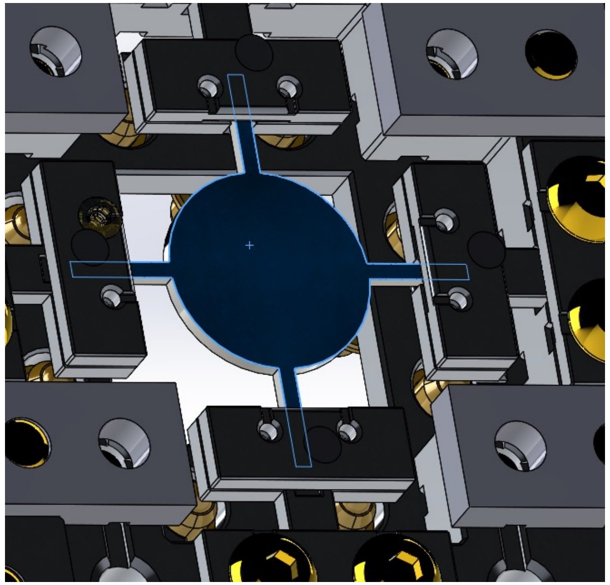
(https://www.emf-corp.com/optical-coatings/dielectric-bragg-mirrors/)

High Frequency Coating Thermal Noise

- Almost all existing work has been done is a quasistatic limit, where the measurement frequency is much less than the first bulk eigenmode
- Have a basic model of how to unify coating and bulk thermal noise

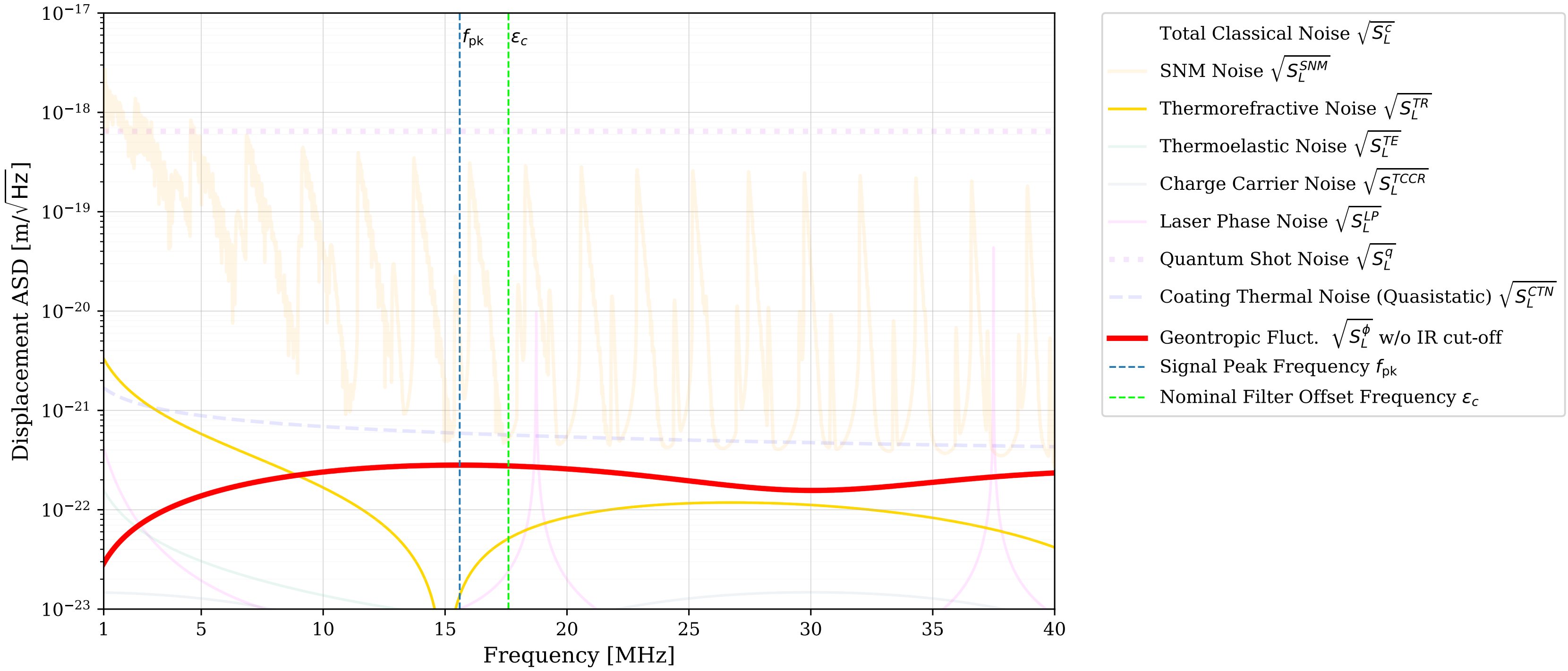
Design Implications

- Bragg Coatings have a low Quality Factor
- This is a fundamental limit on the combined quality factor for the whole mirror
- Goal for us is to hold the optic with less dissipation than in the coating



Thermorefractive Noise

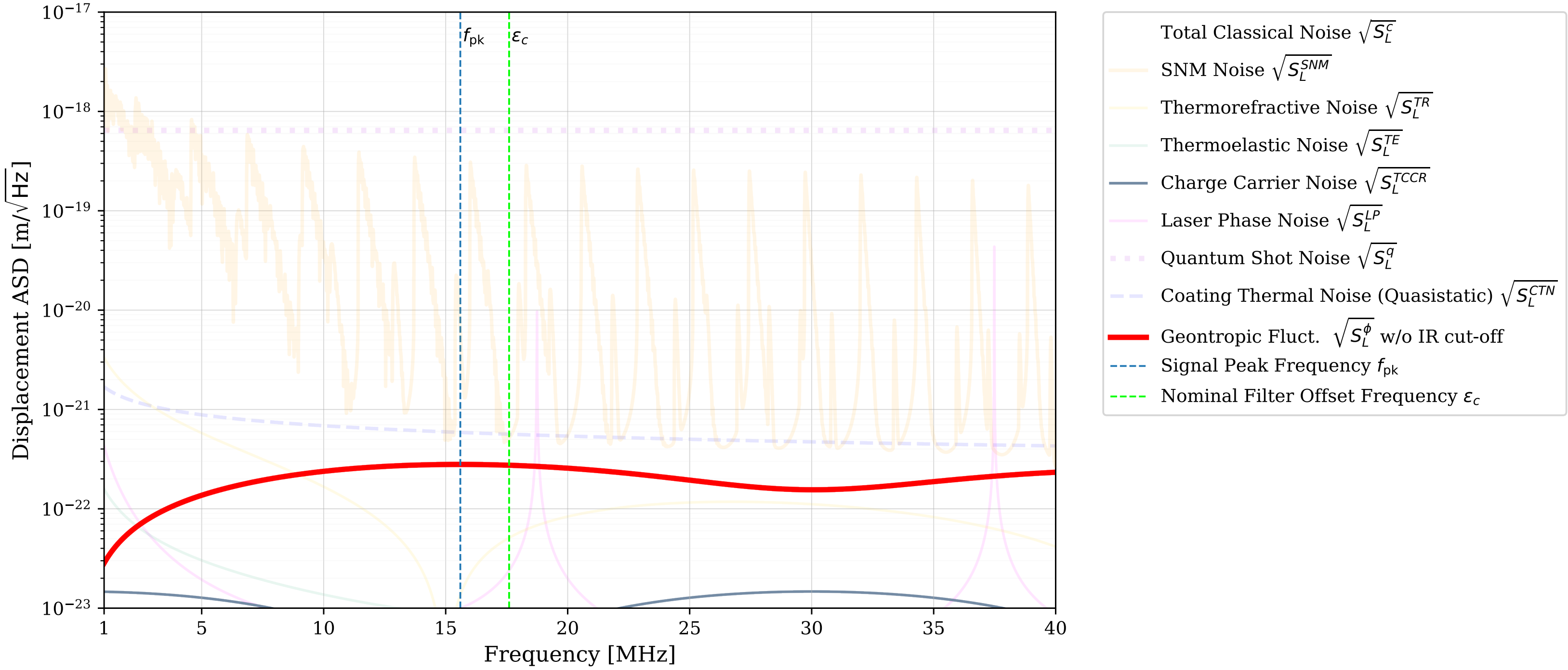
- The index of refraction of silicon, like many materials, is temperature dependent
- The index of refraction changes the optical path length of the transmitted arm
- Optics have microscopic temperature fluctuations due to diffusion
- Noise is present where the light goes through an optic, so just the beamsplitter



$$S_L^{\rm TR}(\Omega) = \frac{4k_B\kappa T^2\beta^2 h}{\pi(C\rho w^2\Omega)^2}$$

Charge Carrier Thermal Noise

- Similar to thermorefractive noise, except here the index of refraction depends on the density of charge carriers, i.e. electrons
- This noise source is constant in frequency up to 200 GHz, although it also experiences the beam splitter transfer function



$$S_L^{\rm TCCR}(\Omega) = \frac{8Dk^2\alpha_e^2 N_0 h}{\pi w^2}\left[\frac{1}{\Omega^2+(4Dk^2+D/l_D^2)^2}\right]$$

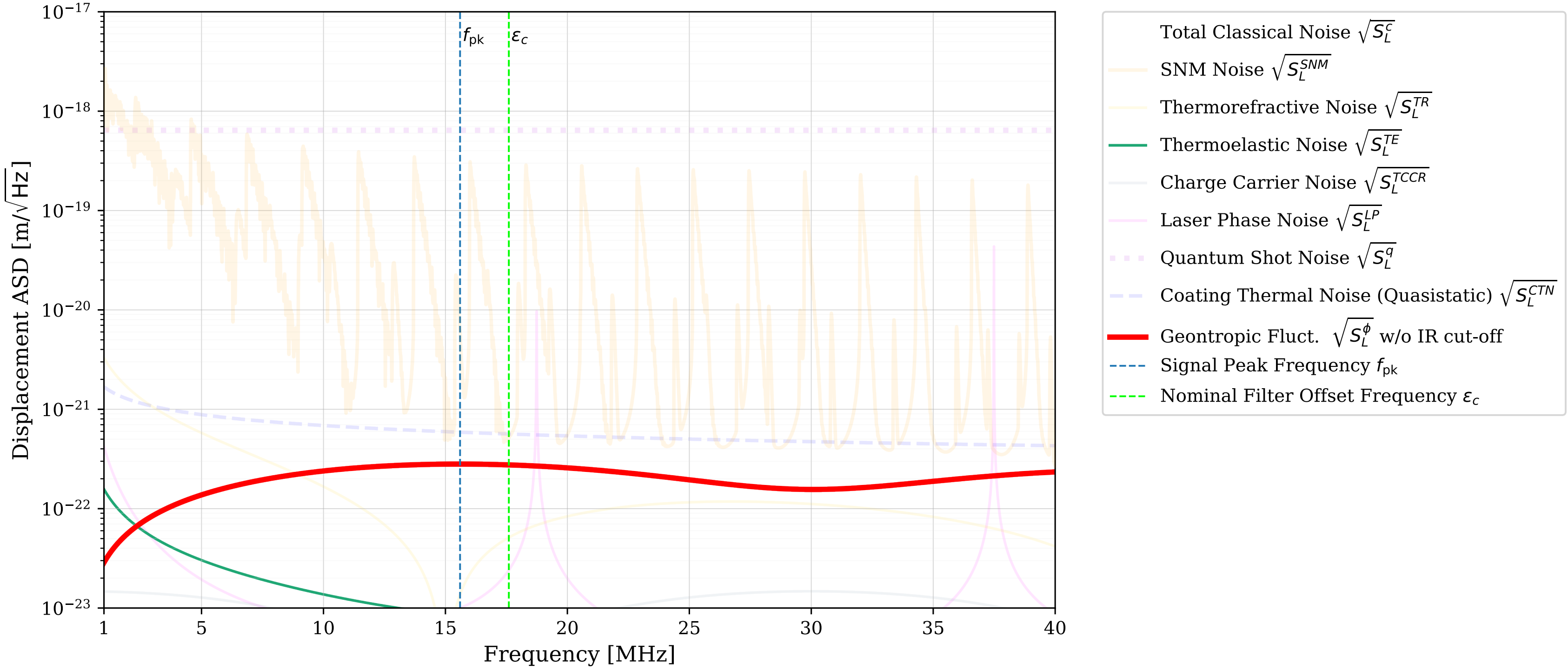
Beam Splitter Transfer Function

- Property inherent to Michelson Interferometer: noise and signals at the beam splitter (very nearly) go away at the FSR (1/light travel time) of the arms
- Can be derived by considering the how light with different phase gets recombined at the beamsplitter

$$H(\Omega) = \cos^2(\frac{\Omega L}{c}) \leq 1$$

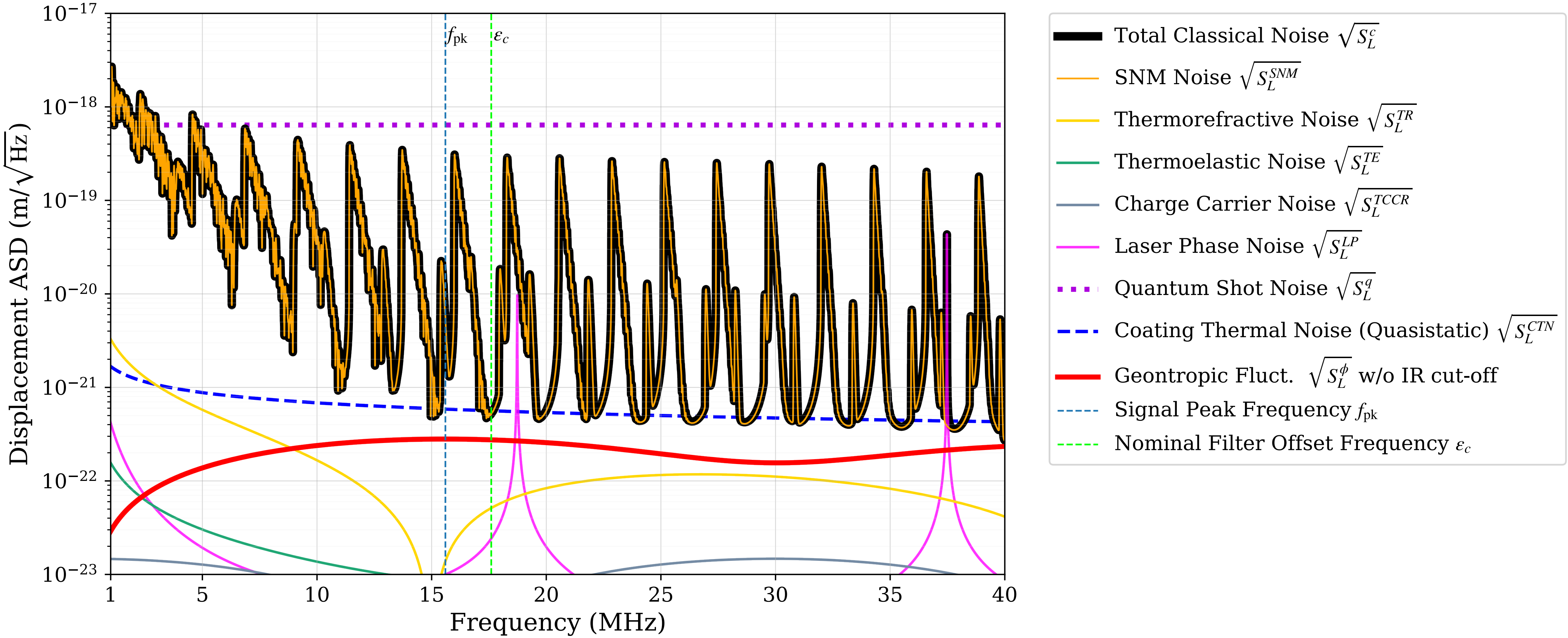
Thermoelastic noise

- Similar to thermorefractive noise, but now we are considering the coefficient of thermal expansion instead of the thermorefractive coefficient
- Present at all optics



$$S_L^{\mathrm{TE}}(\Omega) = \frac{8}{\sqrt{2\pi}} \frac{k_B \kappa T^2 \alpha^2 (1+v_s)^2}{C^2 \rho^2 w^3 \Omega^2}$$

Noise Budget (again)

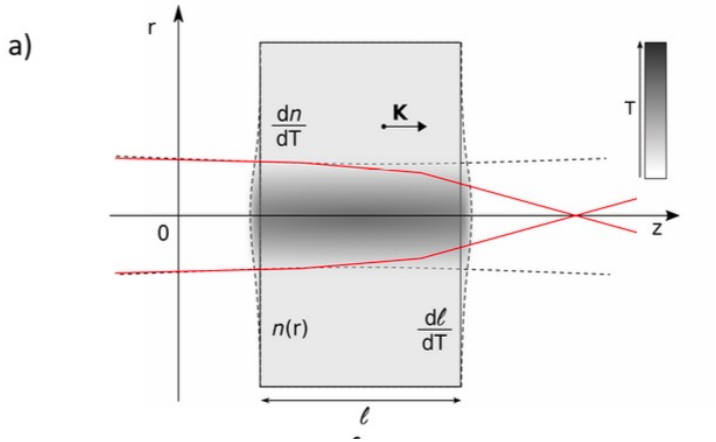


Thermal Lensing

- Not a noise source but an important technical limitation
- The beam splitter absorbs some of the 10 kW going through it
- This introduces a temperature gradient
- Due to the temperature dependence of the index of refraction, this effect lenses light going through the beam splitter
- Need light that is reflected and transmitted to "mode match"

$$\Lambda_{\text{defect}} = 0.067\eta\left(\frac{\beta}{\kappa\lambda}(\Lambda_{\text{Coatings}} + \Lambda_{\text{Body}})P_{\text{BS}}\right)^2$$

$$P_{\text{ASdefect}} = \Lambda_{\text{defect}} P_{\text{BS}}$$



Simonelli et al, 2019

Why Silicon?

- High thermal conductivity
- Stiff
- High Q at 17 MHz
- Lots of R&D previously done
- Good synergy with future detectors
- Drawback #1: need use 1550 nm light
- Drawback #2: semiconductor effects
- Drawback #3: birefringence

Thank you!